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EFFECTS OF CONDENSATION ON THE INTERFACIAL SHEAR STRESS IN LAMINAR FILM CONDENSATION ON A FLAT PLATE

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Laminar film condensation of a saturated vapor in forced flow over a flat plate is analysed. The problem is formulated as an exact boundary-layer solution. From numerical solutions of the governing equations for the ordinary water vapor, Freon-11 and mercury at normal boiling point, it is found that nondimensional values of the film thickness, interfacial shear stress, interfacial velocity and condensation rate are directly affected by the surface temperature of the wall. For the strong condensation case (large $C_P \Delta T/P_r \cdot h_{fg}$), it is found that the magnitude of the interfacial shear stress at the liquid-vapor interphase boundary is approximately equal to the momentum transferred by condensation, i.e., $\tau_i \approx \dot{m}''(U_o - U_i)$.

Key Words : Laminar Film Condensation, Dimensionless Stream Function, Similarity Solution, Interfacial Shear Stress, Adiabatic Shear Stress.

NOMENCLATURE

C_p : Specific heat at constant pressure
E : Modified Jacob number, $C_p(T_s-T_w)/P_r \cdot h_{fg}$
f : Dimensionless liquid stream function, Eq. (5)
F : Dimensionless vapor stream function, Eq. (7)
h_{fg} : Heat of vaporization
k : Thermal conductity
$\dot{m}''$: Mass flow rate at interface(mass condensed per unit time per unit area)
P : Static pressure
Pr : Liquid Prandtl number,$(\mu C_p/k)l$
Re_x : Reynolds number. $U_\infty x/\nu_l$
T : Static temperature
U : Veclocity component in x-direction
V : Veclocity component in y-direction
x : Co-ordinate measuring distance along plate from leading edge
y : Co-ordinate measuring distance normal to the plate
Δ : Thickness of vapor boundary layer or difference
η : Silmilarity variable Eq.(12b)
η_δ : Dimensionless liquid film thickness
θ : Dimensionless temperature, $(T-T_s)/(T_w-T_s)$
μ : Absolute viscosity
ν : Kinematic viscosity
ρ : density
Ψ : Stream function

Subscripts

g : Vapor
i : Liquid vapor interface
l : Liquid
o : Initial value
w : Wall
δ : At the liquid vapor interface or film thickness
∞ : Free stream

Superscripts

: Differentiation with respect to η

1. INTRODUCTION

Research on a subject of laminar film condensation has been focused on natural convection where the fluid motion is generated by gravity forces. The pioneer work was reported by Nusselt(1916), who formulated the problem in terms of simple force and heat balances with the condensate film. The effects of inertia forces, the interfacial shear stress at liquid-vapor interphase and energy convection were not taken into account. There have been a number of improvements in Nusselt's analysis since that time.

The problem of laminar film condensation of a saturated vapor in forced flow over a flat plate is formulated as an exact boundary-layer solution by Koh(1962). He assumed constant fluid properties. Information of the interfacial shear stress at liquid-vapor interphase can be estimated by the exact similarity solutions of the differential governing equations for laminar film condensation on a plate (Koh, 1961, 1962). Lee(1983. 1986) has reported the approximate integral solutions on this subject over a flat plate and at entrance region with assumption of a variable liquid viscosity. The present paper reports the effects of condensation on the shear stress at liquid vapor interphase in forced convection laminar film condensation of a saturated pure substance.

Carpenter and Colburn(1951) first suggested that, as the major force acting on the condensate film is the interfacial shear, total shear stress on the condensate film is affected by shear stress of the vapor, by gravity of the condensate and by the momentum change of the condensing vapor. It was not easy to calculate the interfacial shear stress, because the interfacial shear stress is not independent but coupled with the interfacial mass transfer due to condensation.

Mickley et. al.(1954) and Kinney and Sparrow(1970) reported that suction at the phase boundary can have a significant effect on all of the transport characteristics. Silver(1963~64) and Silver and Wallis(1965~66) have attempted to account for the effect of condensation on interfacial shear stress by using the concept of Reynolds flux in the vapor phase.

Linehan's model(1970) assumed that the interfacial shear stress is augmented by an amount equal to the condensation rate times the average velocity and by adiabatic shear stress.

2. ANALYSIS

2.1 Flow Model

Figure 1 shows a sketch of the physical model and co-ordinate system used for the present study. A mainstream of vapor at a velocity U_o is flowing parallel to the wall direction(x) and velocity distribution is uniform. The vapor is at a saturation temperature T_s. The wall surface temperature T_w is constant and lower than T_s and hence condensa-

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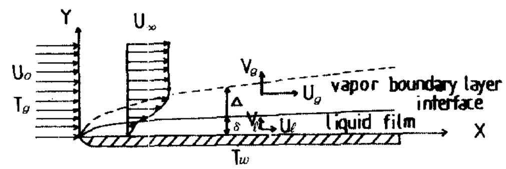

Fig. 1 Physical model and co-ordinate

tion takes place. It is assumed that, in steady state, there exist a wave-free laminar liquid film adjacent to the wall surface. Boundary layer flows are assumed in the immediate neighborhood of the interphases of liquid-vapor and liquid-solid, and potential flow is assumed in the outside region of the vapor boundary layer. The normal direction(y) momentum jump due to mass conversion at the liquid-vapor interface is assumed to be negligible compared with other terms. For laminar two dimensional steady flow, buoyancy and energy dissipation effects are neglected.

2.2 Governing Equations

The basic governing partial differential equations and boundary conditions are as follows.

Liquid film

$$\frac{\partial U_1}{\partial x}+\frac{\partial V_1}{\partial y}=0 \tag{1.a}$$

$$U_1\frac{\partial U_1}{\partial x}+V_1\frac{\partial U_1}{\partial y}=\frac{1}{\rho_1}\left(\frac{\partial \tau_1}{\partial y}-\frac{dP}{dx}\right) \tag{1.b}$$

$$U_1\frac{\partial T_1}{\partial x}+V_1\frac{\partial T_1}{\partial y}=\frac{k_1}{\rho_1 c_1}\frac{\partial^2 T}{\partial y^2} \tag{1.c}$$

Vapor layer

$$\frac{\partial U_g}{\partial x}+\frac{\partial V_g}{\partial y}=0 \tag{2.a}$$

$$U_g\frac{\partial U_g}{\partial x}+V_g\frac{\partial U_g}{\partial y}=\frac{1}{\rho_g}\left(\frac{\partial \tau_g}{\partial y}-\frac{dP}{dx}\right) \tag{2.b}$$

$$U_g\frac{\partial T_g}{\partial x}+V_g\frac{\partial T_g}{\partial y}=\frac{k_g}{\rho_g C_{pg}}\frac{\partial^2 T_g}{\partial y^2} \tag{2.c}$$

Boundary conditions

$$y=0;\ U_1=0, V_1=0, T_1=T_w \tag{3.a}$$

$$y=\delta+\Delta;\ U_g=U_0, T_g=T_s \tag{3.b}$$

$$y=\delta;\ U_1=U_g=U_i \tag{3.c}$$

$$T_1=T_g \tag{3.d}$$

$$\mu\frac{\partial U_1}{\partial y}=\mu_g\frac{\partial U_g}{\partial y}=\tau_i \tag{3.e}$$

$$\dot{m}1''=-\dot{m}''_g=\dot{m}'' \tag{3.f}$$

$$k_1\frac{\partial T_1}{\partial y}=h_{fg}+\left(k_g\frac{\partial Tg}{\partial y}\right) \tag{3.g}$$

where

$$\dot{m}''=\frac{d}{dx}\int_0^\delta \rho_1 U_1 dy \tag{4}$$

By use of the Blasius-type similarity transformation, the partial differential equations can be transformed into a corresponding set of ordinary differential equations. The resulting transformed ordinary differential equations, their boundary conditions and methods of solutions have been given in (Cess, 1960, Kinney et. al. 1970, Lee, 1983) and approximate integral methods of solutions are also given in reference (Koh, 1961,1962). Koh's governing equations as ordinary differential equations are as follow.

Liquid film.

$$f'''+\frac{1}{2}ff''=0 \tag{5}$$

$$\theta''+\frac{1}{2}P_r f\theta'=0 \tag{6}$$

Vapor layer.

$$F'''+\frac{1}{2}FF''=0 \tag{7}$$

where f, F and θ are dimensionless stream function and dimensionless temperature, respectively and prime(′)means differentiation with respect to similarity variable(see Eq. 12a, 12b).

3. INTERFACIAL SHEAR STRESS

3.1 Shear Stress at Liquid-Vapor Interphase

Since the liquid flow is due to the drag force (i.e., interfacial shear stress) exerted by the relatively high speed vapor, and the presence of the liquid motion in turn affects the vapor flow field, this interaction requires the simultaneous consideration of the liquid and vapor layers.

Integrating the momentum equation in the vapor phase over the thickness of the boundary layer from the liquid-vapor interface(δ) to edge of the vapor boundary layer(Δ), the following equation of the interfacial shear stress(τ_i) can be obtained generally,

$$\tau_i=\dot{m}''(U_\infty-U_i)+U_\infty^2\frac{d}{dx}\int_\delta^{\delta+\Delta}\rho_g\frac{U_g}{U_\infty}\left(1-\frac{U_g}{U_\infty}\right)dy-\frac{dP_1}{dx}\Delta \tag{8}$$

where U_∞ is the velocity of potential flow region. If there is no condensation($\dot{m}''=0$) on the flat plate($dP/dx=0$), the first and fourth terms in the right hand side of the above equation become zero and the remained terms are same as the adiabatic shear stress(τ_a)in a vapor phase only.

$$\tau_a=0.332\ \rho_g U_\infty^2\sqrt{\nu_g/U_\infty x} \tag{9}$$

The importance of the first term of the right handside of the equation has been examined and the two-phase condensation problem has been solved numerically for ordinary water vapor at the saturated temperature of 100℃ by using similarity solution and approximate integral methods and also for Freon-II and mercury at normal boiling point.

Once the boundary-layer equations are solved, the values for dimensionless liquid and vapor stream functions, velocity and temperature profiles are available. The condensate flow rate($\dot{m}''$), interfacial velocity, shear stress at the liquid-vapor interphase and heat transfer can then be computed by the following equations and the obtained numerical results(Cess, 1960, Koh, 1961,1962).

Interfacial shear stress(τ_i) by similarity solution.

$$\tau_i=\mu_1 U_0 f''(\eta_\delta)\sqrt{\frac{U_0}{\nu_1 x}} \tag{10}$$

3.2 Dimensionless Condensation Rate

$$\frac{\dot{m}''\sqrt{Re_x}}{\rho_1 U_0}=\frac{f(\eta_\delta)}{2} \tag{11}$$

where

$$f(\eta_1)=\frac{\Psi_1}{\nu_1 U_0 x} \tag{12a}$$

$$\eta_1=\frac{y}{x}\sqrt{U_0 x/\nu_1} \tag{12b}$$

$$\eta_\delta=\frac{\delta}{x}\sqrt{Re_x} \tag{12c}$$

Dimensionless liquid film thickness η_δ is implicitly related to the dimensionless physical group(Jacob number) $C_{pl}(T_s-T_w)/(P_r h_{fg})$ by the following equation:

$$\frac{C_{pl}(T_s-T_w)}{P_r h_{fg}}=-\frac{f(\eta_\delta)}{2\theta'(\eta_\delta)} \tag{13}$$

where

$$\theta(\eta_1)=(T-T_s)/(T_w-T_s) \tag{14}$$

Table 1 Similarity solution of laminar film condensation of water on a flat plate ($T_s=100°C$, $U_o=1m/s$, $P_r=1.76$, $\sqrt{\rho_l\mu_l/\rho_g\mu_g}=190$)

T_w(°C)	η_δ	E×1000	$\frac{U_i}{U_o}\times1000$	$\frac{\dot{m}''\sqrt{Re_x}}{\rho_l U_o}\times1000$	$\frac{\tau_i}{\dot{m}''(U_o-U_i)}$	$\frac{\tau_i}{\tau_a}$
99.9996	0.1	0.004	0.02	0.00	400.070	1.00
99.9729	0.4	0.03	0.07	0.07	25,018	1.03
99.4953	1.0	0.54	0.21	0.54	4.008	1.23
99.0266	1.2	1.03	0.29	0.86	2.785	1.37
98.1978	1.4	1.92	0.39	1.37	2.048	1.60
97.5474	1.5	2.6	0.46	1.74	1.785	1.76
96.625	1.6	3.59	0.56	2.24	1.570	2.00
95.238	1.7	5.06	0.70	2.97	1.382	2.35
92.921	1.8	7.52	0.93	4.17	1.243	2.94
88.090	1.9	12.66	1.39	6.6	1.119	4.18
68.223	2.0	33.78	3.32	16.6	1.024	9.42
22.794	2.05	82.06	7.51	38.7	1.005	20.55
9.677	2.06	96.11	8.66	44.8	1.004	23.49

3.3 Dimensionless Interfacial Velocity

$$U_i=U_o f'(\eta_\delta) \quad (15)$$

Interfacial velocity U_i is dependent on the dimensionless liquid film thickness η_δ(i.e., dependent on tlemperature difference T_s-T_w because η_δ is determined by Jacob number).

From the above equations, condensation rate $\dot{m}''$ and interfacial shear stress τ_i are inversely proportional to the square root of a distance along a flat plate from leading edge(x). However, the ratio of the interfacial shear stress, τ_i and momentum change due to phase conversion, $\dot{m}''(U_o-U_i)$ becomes constant for the given conditions of temperature difference (i,e., Jacob number) or η_δ in the similarity solution.

$$\frac{\tau_i}{\dot{m}''(U_o-U_i)}=\frac{2f''(\eta_\delta)}{f(\eta_\delta)}\frac{U_o}{(U_o-U_i)} \quad (16)$$

Table 1 shows how much the interfacial shear stress τ_i is dependent on the momentum change due to condensation, $\dot{m}''(U_o-U_i)$. For the case of strong condensation (i,e., high Jacob number, or low wall temperature T_w, or large dimensionless film thickness (η_δ), the interfacial shear stress τ_i is approximately same as the momentum change due to phase change $\dot{m}''(U_o-U_i)$. For the case of weak condensation, the interfacial shear stress becomes much larger than momentum change $\dot{m}''(U_o-U_i)$, and values of the interfacial shear stress approaches the adiabatic shear stress(τ_a) which is same as Blasius solution.

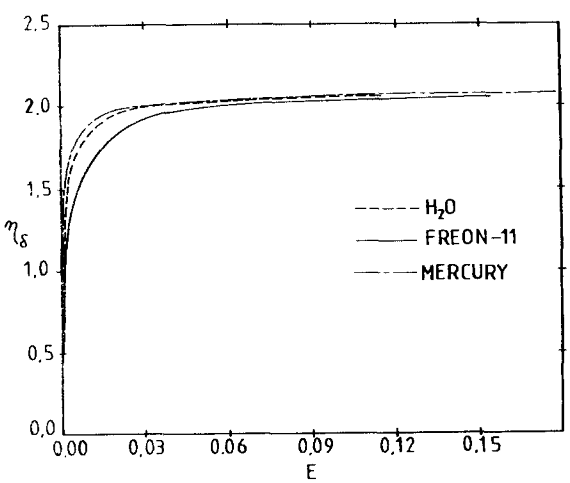

Fig. 2 Physical parameter E and liquid film thickness

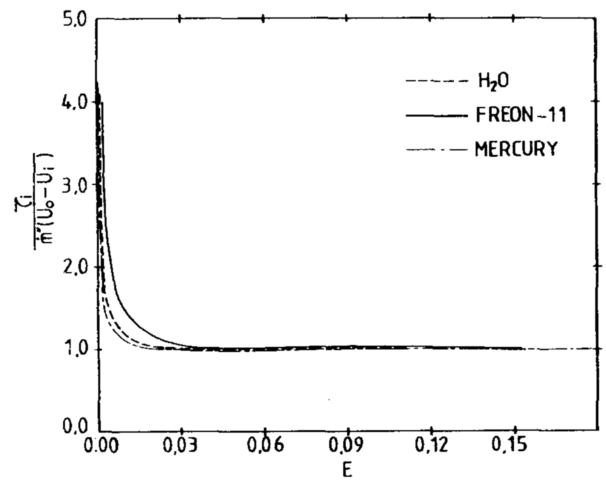

Fig. 3 Interfacial shear stress

WATER, FREON-11, MERCURY CONDENSATION AT NORMAL BOILING POINT

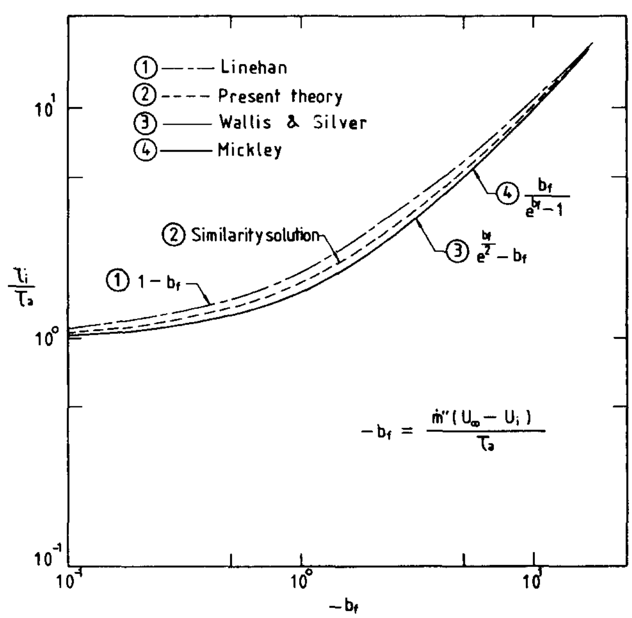

Fig. 4 Comparison of interfacial shear stress models with mass transfer

4. CONCLUSIONS

The solutions of the two-phase boundary-layer equations in laminar film condensation have been investigated in order to find the effect of condensation on the interfacial shear stress at the phase boundary of liquid and vapor flows. The governing differential Eqs. (5,6,7) and boundary conditions involve three parameters Pr, $R=\sqrt{(\rho\mu)_l/(\rho\mu)_g}$ and η_δ (or equivalent value E by Eq. (13)). But solutions are calculated for the saturated water vapor, Freon-11 and mercury at normal boiling point and for various subcooling parameters of $C_{pl}(T_s-T_w)/P_{rl}\cdot h_{fg}$ at 1 atmospheric pressure. For strong condensation(Figs. 2 and 3 show the numberical results for large subcooling parameter E), the interfacial shear stress is approximately determined by the momentum change due to phase conversion $\dot{m}''(U_o-U_i)$, but for weak condensation, the interfacial shear stress approaches to the same value of Blasius solution as the standard boundary layer flow of a vapor phase on a flat plate. It is found that Linehan's expression($\tau_i=\dot{m}''\ U_o+\tau_B$) predicts a larger total interfacial shear stress for strong condensation because exact solutions confirm that $\tau_i=\dot{m}''(U_o-U_i)$ for that case.

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